where $t(w)$ is implicitly specified by the condition $\rho_{\Phi}^{(t)}(Aw, \hat{\phi}) = \delta$ at specified δ and constant temperature level $\hat{\phi}$, under the condition that $w \in W$, a compact specified in advance.

For the limiting case $\beta(u) = \infty$, a similar problem which will not be discussed in detail here was solved in [6].

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APPLICATION OF ITERATIVE REGULARIZATION FOR THE SOLUTION OF INCORRECT INVERSE PROBLEMS

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UDC 536.24

The solution of inverse heat-conduction problems using regularizing gradient algorithms is considered.

Many structures in various engineering fields operate in conditions of intensive and often extremal thermal treatment. The general trend is associated with increase in the number of thermally loaded engineering objects and with increasingly rigorous conditions of thermal loading, with simultaneous increase in reliability and working life and decrease in volume of the material. Questions regarding the maintenance of thermal conditions also occupy an important position in the design and development of technological processes associated with the heating and cooling of materials, for example, in the continuous casting of steel, various methods of heat treatment of metals, glass production, foundry processes, growing high-temperature single crystals from melt, etc.

S. Ordzhonikidze Moscow Aviation Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 53, No. 5, pp. 843-851, November, 1987. Original article submitted March 17, 1987.

0022-0841/87/5305-1335 $12.50

The characteristic features of the thermal conditions of modern thermally loaded structures and technological processes include nonstationarity, nonlinearity, and multidimensionality of the heat-transfer phenomena. These features limit the potential for use of many traditional calculational—theoretical and experimental methods. Therefore, in the last 10-15 years, it has become necessary to develop new approaches to thermophysical and thermotechnical investigations.

Among the high-priority trends, common to various branches of industry, in the development of theoretical and experimental fundamentals for the investigation, maintenance, and modification of the thermal conditions of structures and technological processes is the creation and widespread introduction of experimental and theoretical methods of diagnostics and identification, based on the solution of inverse heat-transfer problems. The resolution of this question, in our view, lays the foundation for the realization of scientifically based methods for thermal experiments and tests and for considerable increase in their information content. This, in turn, accelerates the development and refinement of progressive technologies, and allows new practically inertialess methods of thermophysical measurement to be created for regular and extremal operating conditions of the objects. There is then a possibility of nondestructive monitoring of the thermal-engineering characteristics of structures and technological processes in a real time scale, without disruption of their operating conditions. This approach allows the actually existing effects of nonstationarity and nonlinearity of the heat- and mass-transfer processes to be taken into account, permits experimental investigations that approximate the actual situations as closely as possible or that are conducted directly in the operation of the engineering objects and the industrial realization of the technologies, and also offers the possibility of monitoring and regulating the thermal state of engineering systems. Note that the successful use of this methodology is only possible with a rational combination of physical, technical, and mathematical aspects of the given problem. Therefore, it is necessary to develop experimental and theoretical methods of diagnostics and identification, including the physical and mathematical formulation of inverse-heat-transfer problems, methods and algorithms for their solution, experiment planning, and the corresponding technical necessities for the experimental investigations.

Recently, there has been considerable expansion in the range of formulations of inverse heat-transfer problems used in practice. As well as the most widespread inverse heat-conduction problems (IHP), they also include inverse problems of radiational heat transfer and various formulations of inverse problems of conductive—radiational and conductive—convective type [1-3]. Only the solution of IHP is dealt with below, but it must be noted that the region of application of the given methods and techniques may be considerably broader.

The principal distinguishing feature of IHP is that, as a rule, it is incorrectly formulated. This is because IHP are mathematical models of physically irreversible processes, in which the natural relation between cause and effect is disrupted.

The incorrect formulation of IHP leads to considerable increase in the strictness of requirements on the analysis of the problem, choice of methods for its solution, and the method of analyzing the experimental data. In analyzing IHP, the uniqueness of the solution is of especial importance. The existence and uniqueness of solutions of direct problems is sufficiently obvious from an engineering viewpoint, since direct problems are mathematical models of real physical processes. This correctness is reinforced by rigorous mathematical investigations, which are necessary, ultimately, because any mathematical model is constructed under certain simplifications and assumptions, and is not completely adequate to the real process being described.

In inverse problems, it may usually be assumed, with sufficient confidence on the basis of physical considerations, that a solution exists with accurate initial data, but its uniqueness is not obvious, since the same consequence may follow from different sets of causes. For example, the unique determination of the coefficients C and λ in the heat-conduction equation $CT_\tau = (\lambda T_x)_x$ solely from the data of temperature measurements is impossible, even if these coefficients are constant. Analysis of the conditions of uniqueness of the IHP solution — see [4-6], for example — permits the formulation of requirements imposed on the experimental conditions when the experimental data are assumed to be analyzed by means of the solution of inverse problems. In those cases where the IHP solution is not unique, the regularizing algorithms nevertheless usually allow the a priori estimates of the desired characteristics to be somewhat refined, if it is ensured that the approximations to the normal solution obtained are close to the initial approximation of the IHP solution, which is known to be closer to the true solution than the a priori estimate.

The incorrectness of the IHP due to instability and sometimes also nonuniqueness of the solution means that regular methods must be used for their solution. All methods of solving IHP are arbitrarily divided into universal and problem-oriented methods. Universal methods include algorithms which make use only of information of the most general character and are suitable for the solution of a broad class of operator equations, for example, algorithms based on the Tikhonov variational approach to the construction of regularizing operators [7] and regularizing gradient algorithms [8, 9]. The problem-oriented methods include algorithms in which the specific features of the problem to be solved are taken significantly into account – for example, direct methods [10], "thin-wall" methods [11], algorithms based on the quasisolution method, etc. Problem-oriented algorithms have a considerably narrower region of applicability usually imposing fairly strict requirements on the experimental conditions, but within their region of applicability they are faster than universal algorithms as a rule, and sometimes also more accurate. Therefore, it is desirable for researchers to have a complex of programs – both universal and problem-oriented – at their disposal.

An indissoluble component of thermal investigations using inverse-problem methods is the computational experiment, especially in conducting expensive full-scale tests. Extensive modeling is also necessary here at the stage of designing a thermal experiment (since the accuracy of solution of inverse problems may depend significantly, for example, on the disposition of the temperature sensors [12]) and at the stage of preparing the experimental data for analysis, as well as in the analysis itself. In preparing experimental data for analysis, a computational experiment may be used to choose the most appropriate algorithm for IHP solution in the given specific case, to choose the parameters of the computational algorithm, to estimate the influence of indeterminacies in the mathematical model and its parameters, the measurement errors, etc. A priori concepts regarding the character of the desired solution are used here, on the basis of physical considerations, previous experimental calculations, or calculations using more complex mathematical models. Programs for IHP solution are used at this stage in modeling conditions, when the corresponding measurable characteristics are calculated from the presumed values of the desired characteristics on the basis of the solution of direct problems. These data include noise, modeling the errors of the measurement and recording complex. Then, from the "initial data" obtained, the corresponding IHP is solved, and the results are compared with the presumed values.

After analyzing the real experimental data, careful modeling must also be undertaken in order to estimate the reliability and accuracy of the results obtained.

Thus, developed software in the form of bundles of applied programs permitting analysis of the experimental data and also broad modeling of heat-transfer processes must be available to researchers.

Programs for IHP solution on the basis of universal algorithms form a basic component of the given bundles of applied programs. Regularizing gradient algorithms recommend themselves highly for use as such algorithms. Their structure is illustrated for the example of the method of fastest descent. Consider the operator equation

$$Au = f,\ A: U \to F, \tag{1}$$

where A is a Fréchet-differentiable operator; U, F are Hilbertian spaces. Suppose that, instead of the accurate initial data $\{f, A\}$, approximate data $\{f_\delta, A_h\}$ are specified, where $\|f_\delta - f\|_F \leq \delta$ and A_h is some operator approximating the initial operator.

The sequence of approximations in the method of fastest descent for the approximate initial data takes the form

$$u_{n+1} = u_n - \beta_n J'u_n,\ n = 0,\ 1,\ 2,\ \ldots, \tag{2}$$

where $J'u_n = (A'_h)^*(A_h u_n - f_\delta)$ is the gradient of the discrepancy functional $J(u) = (1/2) \cdot \|A_h u - f_\delta\|_F^2$; A_h' is the Fréchet derivative of the operator A_h at point u_n; $(A_h')^*$ is the conjugate operator. If A_h is a linear operator, then $A_h' = A_h$, and the descent step β_n is calculated from the formula: $\beta_n = \|J'u_n\|_U^2/\|A_h J'u_n\|_F^2$, which is obtained from the condition of a minimum of the functional J(u) in the given iteration. In nonlinear IHP, sufficiently good results are often obtained using a linear estimate for the descent step [13]: $\beta_n = \|J'u_n\|_U^2/\|A_h' J'u_n\|_F^2$. More accurate algorithms may also be used for the calculation of β_n in the nonlinear case, but, since they require repeated solution of the direct problem, the resulting gain in rate of decrease of the function rarely justifies the additional computational time required.

For linear problems, the method of fastest descent with cessation according to the discrepancy criterion is a regularizing algorithm [8, 9], and in the case of a nonunique solution of Eq. (1) stability of the approximations obtained is ensured relative to normal solution of Eq. (1), i.e., the solution closest to the initial approximation u_0. If the errors of the approximation may be neglected in comparison with the errors on the right-hand side of Eq. (1), the cessation condition takes the form

$$N = N_H = \min_n \left\{ n : \frac{\Delta_{n+1}^2 + \Delta_n^2}{2\Delta_n} < c\delta \right\}, \tag{3}$$

where $\Delta_n = \|A_h u_n - f_\delta\|_F$, $c > 1$ is some constant introduced so as to ensure that Eq. (3) holds for finite N. The inequality $\Delta_{N+1} < c\delta < \Delta_{N-1}$ holds here. In the case of IHP solution, as shown by calculations, approximations with numbers close to N_H change slowly and the process may be stopped in the first iteration, for which $\Delta_N \leq \delta$.

Computational experiments for different nonlinear boundary and coefficient IHP have demonstrated the high efficiency of this approach also in the nonlinear case.

Gradient algorithms are very convenient from the viewpoint of practical realization, since the values of the derivative operator and its conjugate operator are found from the solution of boundary problems that are of the same type as the initial direct problem; therefore, a good program for solution of the direct problem is sufficient for calculation using Eq. (2). In gradient algorithms, a priori information on the desired quantity may be taken into account sufficiently simply [14-16]. Another advantage is the possibility of using the results of several measurements in sufficiently arbitrary time periods. This may be shown by an example. Consider a boundary IHP in a region with a movable gradient

$$C(T)T_\tau = (\lambda(T)T_x)_x,\ X_0(\tau) < x < b,\ 0 < \tau \leqslant \tau_m, \tag{4}$$

$$T(x,\ 0) = T_0(x), \tag{5}$$

$$-\lambda T_x|_{x=X_0(\tau)} = u(\tau), \tag{6}$$

$$-\lambda T_x|_{x=b} = q_0(\tau). \tag{7}$$

Unique determination of the unknown heat-flux density $u(\tau)$ with a specified law of motion of the boundary $X_0(\tau)$ entails having at least one temperature measurement at an internal point of the body on the whole interval $[0, \tau_m]$: $f_1(\tau) = T(d_1, \tau)$, $X_0(\tau_m) \leq d_1 \leq b$; for the sake of simplicity, it is assumed that $X_0(\tau)$ increases monotonically. However, as is known, the influence of measurement errors increases as the distance of the measuring device from the surface with the given boundary conditions increases. Therefore, to reduce the influence of measurement errors, it is expedient to place additional temperature sensors at the points $d_2 > d_3 \ldots > d_k \geq X_0(0)$ closer to the heated surface; $d_2 < X_0(\tau_m)$. Information from these sensors is only obtained, ultimately, on segments $[0, \tau_i]$, $i = \overline{2, k}$, where τ_i are determined, for example, from the condition $X_0(\tau_i) = d_i$ (or from the condition that limiting temperature values for the given sensors are attained). In this case, the values of the operator A are vector functions $f(\tau) = \{f_1(\tau), \ldots, f_k(\tau)\}$, $f_i(\tau) = T(d_i, \tau)$, $\tau \in [0, \tau_i)$, $i = \overline{1, k}$, $\tau_1 = \tau_m$. Final formalization of the problem in the form of Eq. (1) demands a choice of the spaces U and F. The Hilbertian space $L_2[0, \tau_m]$ with norm $\|u\|_U^2 = \int_0^{\tau_m} u^2(\tau)d\tau$ is chosen as U and the Hilbertian space $L_2^k[0, \tau_m]$ with the norm

$$\|f\|_F^2 = \sum_{i=1}^{k} \int_0^{\tau_m} \rho_i(\tau) f_i^2(\tau)\, d\tau$$

as F; here $\rho_i(\tau) \geq 0$ is some weighting function. To derive the conjugate problem, it is expedient to assume that $f_i(\tau)$ is specified on the whole segment $[0, \tau_m]$ and the lack of information when $\tau > \tau_i$ may be taken into account by setting the corresponding weighting function equal to zero: $\rho_i(\tau) = 0$, $\tau > \tau_i$. The specific form of the function $\rho_i(\tau)$ is chosen taking account of the relative measurement error at points d_i. Preliminary modeling is also useful here. In the simplest case, it may be assumed that

$$\rho_i(\tau) = \begin{cases} 1, & \tau \in [0, \tau_i], \\ 0, & \tau > \tau_i. \end{cases}$$

The value of the operator A may be regarded as a vector

$$Au = \{A_1u, \ldots, A_ku\}, \; A_iu = f_i,$$

$$A_i : L_2[0, \tau_m] \to L_2[0, \tau_m].$$

The derivative A' of operator A at point u is determined from the solution of a linear heat-conduction boundary problem

$$(C(T(x, \tau))\theta)_\tau = (\lambda(T(x, \tau))\theta)_{xx}, \; X_0(\tau) < x < b, \; 0 < \tau \leqslant \tau_m, \tag{8}$$

$$\theta(x, 0) = 0, \tag{9}$$

$$-(\lambda(T(x, \tau))\theta)_x|_{x=X_0(\tau)} = \Delta u(\tau), \tag{10}$$

$$-(\lambda(T(x, \tau))\theta)_x|_{x=b} = 0, \tag{11}$$

$$(A_i'\Delta u)(\tau) = \theta(d_i, \tau), \; i = \overline{1, k}, \tag{12}$$

where $T(x, \tau)$ is the solution of the problem in Eqs. (4)-(7) for the specified function $u(\tau)$ (the point at which the derivative A' is sought). Here $A'\Delta u = \{A_1'\Delta u, \ldots, A_k'\Delta u\}$.

The value of the conjugate operator $(A')^*\phi$, $\phi = \{\phi_1, \ldots, \phi_k\}$ is found from the solution of the conjugate boundary problem

$$C(T(x, \tau))\psi_\tau + \lambda(T(x, \tau))\psi_{xx} = 0, \; X_0(\tau) < x < b, \; 0 \leqslant \tau < \tau_m, \tag{13}$$

$$\psi(x, \tau_m) = 0, \; X_0(\tau_m) \leqslant x \leqslant b, \tag{14}$$

$$-\lambda(T(x, \tau))\psi_x + C(T(x, \tau))X_0'(\tau)\psi|_{x=X_0(\tau)} = 0, \tag{15}$$

$$-\lambda(T(x, \tau))\psi_x|_{x=b} = 0, \tag{16}$$

$$\psi(d_i - 0, \tau) = \psi(d_i + 0, \tau), \; i = \overline{1, k}, \tag{17}$$

$$-\lambda(T(x, \tau))\psi_x|_{x=d_i+0} + \lambda(T(x, \tau))\psi_x|_{x=d_i-0} = \rho_i(\tau)\varphi_i(\tau), \; i = \overline{1, k}, \tag{18}$$

$$((A')^*\varphi)(\tau) = \psi(X_0(\tau), \tau). \tag{19}$$

The conditions in Eq. (17) express the continuity of the function ψ at points d_i, while Eq. (18) expresses the discontinuity of $-\lambda\psi_x$ at points d_i with a discontinuity of magnitude $\rho_i(\tau)\phi_i(\tau)$.

Thus, at each iteration in the method of fastest descent, for the approximation $u_{n+1}(\tau)$ obtained, the direct problem is solved, and the coefficients of the conjugate problem in Eqs. (13)-(19) and $\phi_i(\tau) = T(d_i, \tau, u_n) - f_i(\tau)$ in Eq. (18) are calculated. Then Eq. (19) gives the value of the gradient $(J'u_n)(\tau) = \psi(X_0(\tau), \tau)$. To determine the linear estimate of the descent step, the problem in Eqs. (8)-(12) is solved with $\Delta u(\tau) = (J'u_n)(\tau)$, and the corresponding coefficients are calculated using $T(x, \tau, u_n)$. Here

$$\beta_n = \frac{\int_0^{\tau_m} \psi^2(X_0(\tau), \tau)\, d\tau}{\sum_{i=1}^{k} \int_0^{\tau_m} \rho_i(\tau)\theta^2(d_i, \tau)\, d\tau}.$$

In the presence of a priori information on the smoothness of the desired solution, it may be taken into account using one of the methods proposed in [14-16]. The iterative process may be halted by means of the discrepancy criterion.

Gradient algorithms allow the measurement errors to be taken into account also in the case when the boundary conditions in the direct problem are determined by experiment (and not specified accurately). This is demonstrated for the example of the following boundary IHP

$$C(T)T_\tau = (\lambda(T)T_x)_x,\ 0<x<b,\ 0<\tau\leqslant\tau_m, \tag{20}$$

$$T(x,\ 0) = T_0(x), \tag{21}$$

$$-\lambda T_x|_{x=0} = u_1(\tau), \tag{22}$$

$$T(b,\ \tau) = g(\tau), \tag{23}$$

$$(Au)(\tau) = T(d,\ \tau) = f(\tau),\quad d\in(0,\ b). \tag{24}$$

Suppose that the temperatures $g(\tau)$ and $f(\tau)$ are experimental functions and thus contain random errors. The function $g(\tau)$ appears in the condition of uniqueness of the boundary problem in Eqs. (20)-(23), i.e., in the definition of the operator A, and $f(\tau)$ may be regarded as the right-hand side in Eq. (1).

In the linear case, when $C = C(x,\ \tau)$ and $\lambda = \lambda(x,\ \tau)$, the influence of random errors in the function $g(\tau)$ may be taken into account on passing to the integral analog of the problem in Eqs. (20)-(24) using the superposition principle

$$\int_0^t K_1(t,\ \tau)\,u_1(\tau)\,d\tau = f_1(t),$$

where

$$f_1(t) = f(t) - \int_0^b K_0(t,\ x,\ \xi)\,T_0(\xi)\,d\xi - \int_0^t K_2(t,\ \tau)\,g(\tau)\,d\tau.$$

This entails estimating the error introduced by the last integral term on the right-hand side of $f_1(\tau)$. This error decreases with increase in the distance between points d and b.

In the nonlinear case, the superposition principle does not hold, and therefore another approach is used. The temperature $T(b,\ \tau) = u_2(\tau)$ is included in the functions to be determined, i.e., it is assumed that it is required to determine the functions $u_1(\tau)$, $u_2(\tau)$ from the specified (with an error) functions $f(\tau)$, $g(\tau)$.

To formalize the problem in the form in Eq. (1), the space U of vector functions $u = \{u_1,\ u_2\}$ with norm $\|u\|_U^2 = \|u_1\|^2_{L_2[0,\tau_m]} + \rho\|u_2\|^2_{L_2[0,\tau_m]}$, $\rho > 0$ is introduced, as well as the space F of vector functions $\bar{f} = \{f,\ g\}$ with norm $\|\bar{f}\|_F^2 = \|f\|^2_{L_2[0,\tau_m]} + \gamma\|g\|^2_{L_2[0,\tau_m]}$, $\gamma > 0$. Then the given IHP takes the form

$$\bar{A}u = \bar{f},\quad \bar{A}u = \{T(d,\ \tau,\ u),\ u_2(\tau)\}, \tag{25}$$

where $T(d,\ \tau,\ u)$ is found from the solution of Eqs. (20)-(23), when $g(\tau)$ in Eq. (24) is replaced by its estimate $u_2(\tau)$. The derivative $\bar{A}'\Delta u$ is found using a linear boundary problem

$$(C(T(x,\ \tau))\,\theta)_\tau = (\lambda(T(x,\ \tau))\,\theta)_{xx}, \tag{26}$$

$$\theta(x,\ 0) = 0, \tag{27}$$

$$(-\lambda\theta)_x|_{x=0} = \Delta u_1(\tau), \tag{28}$$

$$\theta(b,\ \tau) = \Delta u_2(\tau). \tag{29}$$

Here $\bar{A}'\Delta u = \{\theta(d,\ \tau),\ \Delta u_2(\tau)\} = \{\theta_1(d,\ \tau) + \theta_2(d,\ \tau),\ \Delta u_2(\tau)\} = \{A_1'\Delta u_1 + A_2'\Delta u_2,\ \Delta u_2\}$ where $\theta_1(d,\ \tau)$ is the solution of Eqs. (26)-(29) when $\Delta u_2(\tau) \equiv 0$ and $\theta_2(d,\ \tau)$ is the corresponding solution when $\Delta u_1(\tau) \equiv 0$.

To derive the representation of the conjugate operator $(\bar{A}')^*$ in accordance with this definition, the identity

$$(\Delta u,\ \Delta u^*)_U = (\bar{A}'\Delta u,\ \varphi)_F$$

is written for any $\phi = \{\phi_1,\ \phi_2\}\in F$ and any $\Delta u = \{\Delta u_1,\ \Delta u_2\}\in U$, where $\Delta u^* = (\bar{A}')^*\phi$ is the value of the conjugate operator. Hence

$$(\Delta u, \Delta u^*)_U = (\Delta u_1, \Delta u_1^*)_{L_2} + \rho(\Delta u_2, \Delta u_2^*)_{L_2} =$$

$$= (A_1'\Delta u_1 + A_2'\Delta u_2, \varphi_1)_{L_2} + \gamma(\Delta u_2, \varphi_2)_{L_2} = (\Delta u_1, A_1'^*\varphi_1)_{L_2} + (\Delta u_2, A_2'^*\varphi_1 + \gamma\varphi_2)_{L_2}.$$

Consequently

$$(\bar{A}')^*\varphi = \left\{A_1'^*\varphi_1, \frac{1}{\rho}(A_2'^*\varphi_1 + \gamma\varphi_2)\right\}.$$

The values of the conjugate operators $A_1'^*$ and $A_2'^*$ are found from the solution of the same conjugate boundary problem

$$C(T(x, \tau))\psi_\tau + \lambda(T(x, \tau))\psi_{xx} = 0, \quad (30)$$

$$\psi(x, \tau_m) = 0, \quad (31)$$

$$-\psi_x|_{x=0} = 0, \quad (32)$$

$$\psi(b, \tau) = 0, \quad (33)$$

$$\psi(d-0, \tau) = \psi(d+0, \tau), \quad (34)$$

$$-\lambda\psi_x|_{x=d+0} + \lambda\psi_x|_{x=d-0} = \varphi_1(\tau), \quad (35)$$

but from different formulas

$$A_1'^*\varphi_1 = \psi(0, \tau), \quad A_2'^*\varphi_2 = -\lambda\psi_x|_{x=b}.$$

The iterative algorithm in Eq. (3) takes the following form in this case

$$u_1^{n+1}(\tau) = u_1^n(\tau) - \beta_n\psi^n(0, \tau) = u_1^n(\tau) - \beta_n\Delta u_1^n(\tau),$$

$$u_2^{n+1}(\tau) = u_2^n(\tau) - \frac{\beta_n}{\rho}[-\lambda\psi_x^n|_{x=b} + \gamma(u_2^n(\tau) - g(\tau))] = u_2^n(\tau) - \beta_n\Delta u_2^n(\tau),$$

where $\psi^n(x, \tau)$ is the solution of Eqs. (30)-(35) when $\phi_1(\tau) = T(d, \tau, u_n) - f(\tau)$ ($T(x, \tau, u_n)$ is the solution of Eqs. (20)-(23) when $u_n = \{u_1^n, u_2^n\}$).

The linear estimate for the descent step β_n is calculated in the given case from the formula

$$\beta_n = \frac{\int_0^{\tau_m}(\Delta u_1^n)^2 d\tau + \rho\int_0^{\tau_m}(\Delta u_2^n)^2 d\tau}{\int_0^{\tau_m}\theta^2(d, \tau)d\tau + \gamma\int_0^{\tau_m}(\Delta u_2^n)^2 d\tau},$$

where $\theta(d, \tau)$ is the solution of Eqs. (26)-(29) when Δu_1^n and Δu_2^n are substituted into Eqs. (28) and (29), respectively.

The coefficients of Eqs. (26)-(29) and (30)-(35) at each iteration are calculated using the temperature field $T(x, \tau, u_n)$.

The iterative process must be terminated by the condition

$$\int_0^{\tau_m}[T(d, \tau, u_n) - f(\tau)]^2 d\tau + \gamma\int_0^{\tau_m}(u_2^n)d\tau \approx \delta_f^2 + \gamma\delta_g^2,$$

where δ_f, δ_g are the mean square errors in specifying the functions $f(\tau)$ and $g(\tau)$, respectively.

The weighting factors ρ and γ may be chosen by preliminary modeling or by using a modified method of fastest descent, with individual choice of the descent step from each component of the descent direction in accordance with the procedure outlined in [17]. Then $\rho = \gamma = 1$ may be assumed. Any smooth function sufficiently close to $g(\tau)$ may be taken as the initial approximation with respect to the second component $u_2^0(\tau)$. Within the framework of this algorithm a priori information on the smoothness of the desired functions may be taken into account.

This approach to the solution of IHP, when the boundary conditions are specified with errors, should be applied, ultimately, only when the distance between points b and d is sufficiently small and the errors in specifying $g(\tau)$ cannot be neglected.

Note, in conclusion, that the methodology of iterative regularization of incorrect problems developed in the last ten years has a broad spectrum of applications in the diagnostics and identification of heat- and mass-transfer processes. This methodology may also have useful applications in solving many inverse problems of mathematical physics arising in other fields of scientific research and engineering application.

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SURVEYS

SOME PROBLEMS OF MASS EXHANGE IN MAGNETIC SUSPENSIONS AND COLLOIDS

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UDC 537.84:532.529.2:532.75

Recently, the attention of specialists has been drawn to different aspects of the interaction of a magnetic field with dispersive magnetized media. A new direction in technology has been developed successfully, the magnetic separation of dispersive materials. Studies have been conducted on the extraction of magnetic components from a nonmagnetic liquid [1-3] and weakly magnetic materials from a magnetic liquid [4-6]. High-gradient magnetic separation of weakly magnetic microparticles has also been intensively studied [7]. Finally, nanoparticles of magnetic liquids are separated magnetically [8]. Magnetophoretic transport of colloidal particles influences the stability of magnetic liquids as well as the working capacity of many technical devices using magnetic liquids (magnetoliquid seals, vibration dampers, printing units with magnetic liquids, etc. [9]). Below we give a concise review of studies on the mass transport of particles in magnetic colloids and suspensions, conducted at the Institute of Physics of the Academy of Sciences of the Latvian SSR.

1. Magnetophoresis of Particles in a Viscous Liquid. The physical basis for the mass transport of particles in a magnetic field is the magnetophoretic force

$$\mathbf{F}_m = V\mu_0 (\mathbf{M}\nabla)\,\mathbf{H}, \tag{1}$$

which is determined by the magnetization of the particle M = M(H) and its volume V. Calculation of the force (1) in the general case of nonequilibrium magnetization is a complicated problem. The quasistationary case $M = \kappa H$ is the easiest one to solve. For this case

Institute of Physics, Academy of Sciences of the Latvian SSR, Riga. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 53, No. 5, pp. 852-860, November, 1987. Original article submitted February 27, 1987.

 0022-0841/87/5305-1342 $12.50